The structure and effectiveness of the Polish tax system
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The optimal marginal labor income tax

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The problem

Following Mirrlees (1971):

$$\max_{\tau()} W = \int_{a_L}^{a_H} G(U_a) f(a)\, da$$

subject to

$$U_a = u\big(y_a - \tau(y_a)\big) - v\left(\frac{y_a}{a}\right)$$

$$u'\big(y_a - \tau(y_a)\big)\big(1 - \tau'(y_a)\big) = \frac{1}{a} v'\left(\frac{y_a}{a}\right)$$

$$R = \int_{a_L}^{a_H} \tau(y_a) f(a)\, da$$

where:

- a – agent's productivity;
- f – PDF;
- G – a concave function of preferences of the policy maker;
- U_a – agent's realized utility;
- y_a – agent's labor income;
- τ – tax function;

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Reformulating the problem

Following Saez (2001):

$$\max_{c,y} u(c, y)$$

subject to

$$c = (1 - \tau)y + m$$

where:

- c – agent's consumption;
- m – government transfer;
- y – labor income;
- τ – tax rate;

The response is

$$(1 - \tau)u_c + u_y = 0$$

which defines implicitly a Marshallian (uncompensated) earnings supply function $y = y(1 - \tau, m)$.

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The effects of an increase in the marginal tax rate

Suppose $d\tau > 0$ for $y > y^*$. Then the total change in R is $R_s + R_d$, where

$$R_s = n(\bar{y} - y^*)\, d\tau \quad \text{and} \quad R_d = n\tau\, dy$$

n – number of the top earners;
$\bar{y}$ – average income (over y^*).
Totally differentiating

$$dy = -\frac{\partial y}{\partial(1-\tau)} d\tau + \frac{\partial y}{\partial m} dm$$

Let

$$\eta^u = \frac{1-\tau}{y} \frac{\partial y}{\partial(1-\tau)} \quad \text{and} \quad \eta = (1-\tau) \frac{\partial y}{\partial m}$$

denote the uncompensated elasticity of income earned and the income effect, respectively. Since $dm = y^*\, d\tau$, then

$$R_d = -n(\overline{\eta^u} \bar{y} - \bar{\eta} y^*) \frac{\tau\, d\tau}{1-\tau}$$

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The optimal tax

The envelope theorem implies that the total welfare loss due to the small variation in τ, experienced by the top earners, is

$$\bar{g}R_s$$

where $\bar{g} \geq 0$ is the ratio of social marginal utility for the top earners to the marginal value of public funds for the government.

Then, the tax is optimal only when

$$R_s + R_d = \bar{g}R_s$$

which implies

$$\tau = \frac{1}{1 + \dfrac{\overline{\eta^u}\dfrac{\bar{y}}{y^*} - \bar{\eta}}{(1-\bar{g})\left(\dfrac{\bar{y}}{y^*} - 1\right)}}$$

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The optimal tax

If PDF of income distribution takes the form of a Pareto distribution

$$P(income > y) = \frac{A}{y^a}$$

where A is a constant, then $\frac{\bar{y}}{y^*} = \frac{a}{a-1}$. Thus

$$\tau = \frac{1 - \bar{g}}{1 - \bar{g} + \overline{\eta^u} a - \bar{\eta}(a-1)}$$

If $\eta = 0$ and $\bar{g} \approx 0$

$$\tau = \frac{1}{1 + \overline{\eta^u} a}$$

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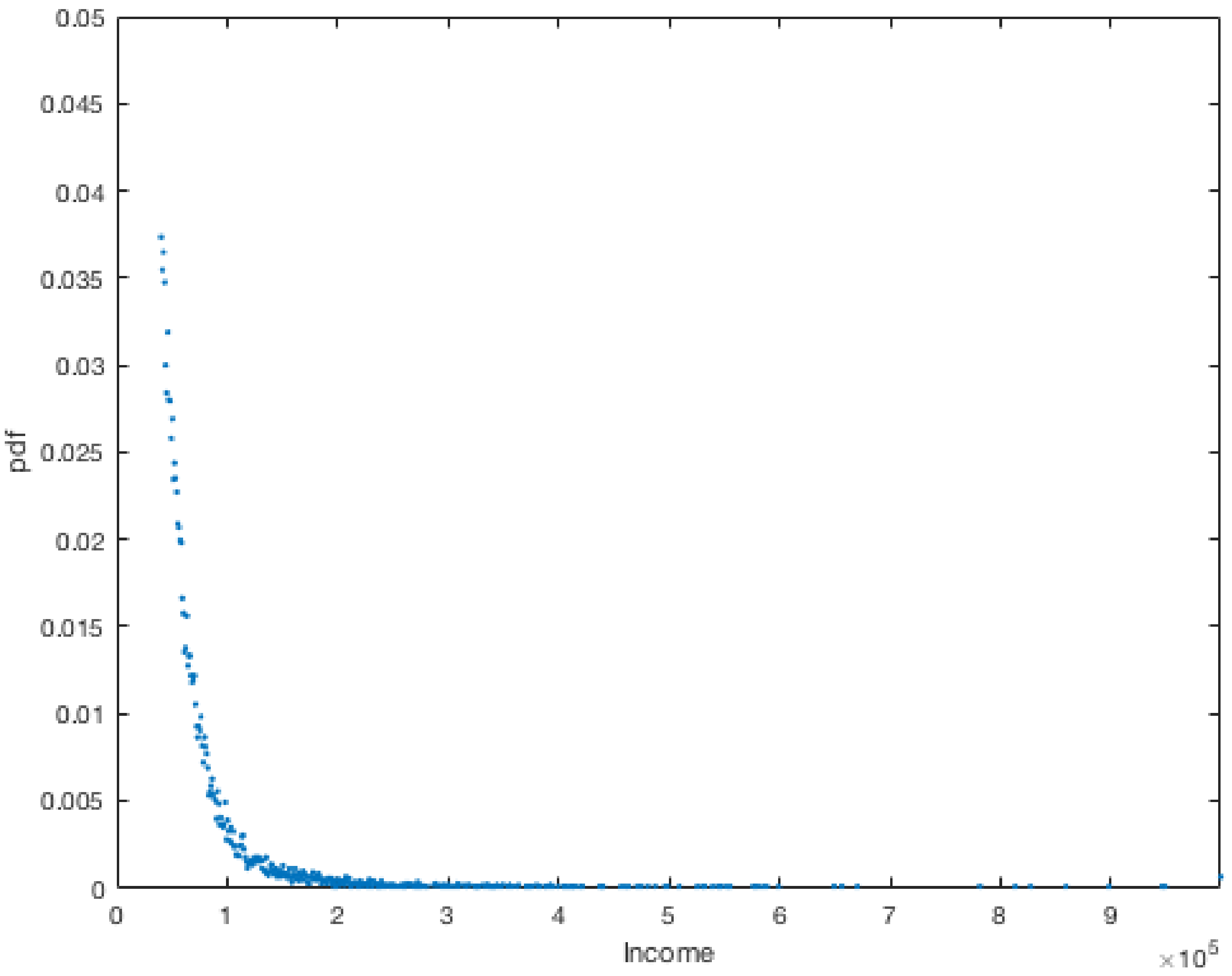
The income distribution in Poland



sample of 50,000 taxpayers (MF, 2016)

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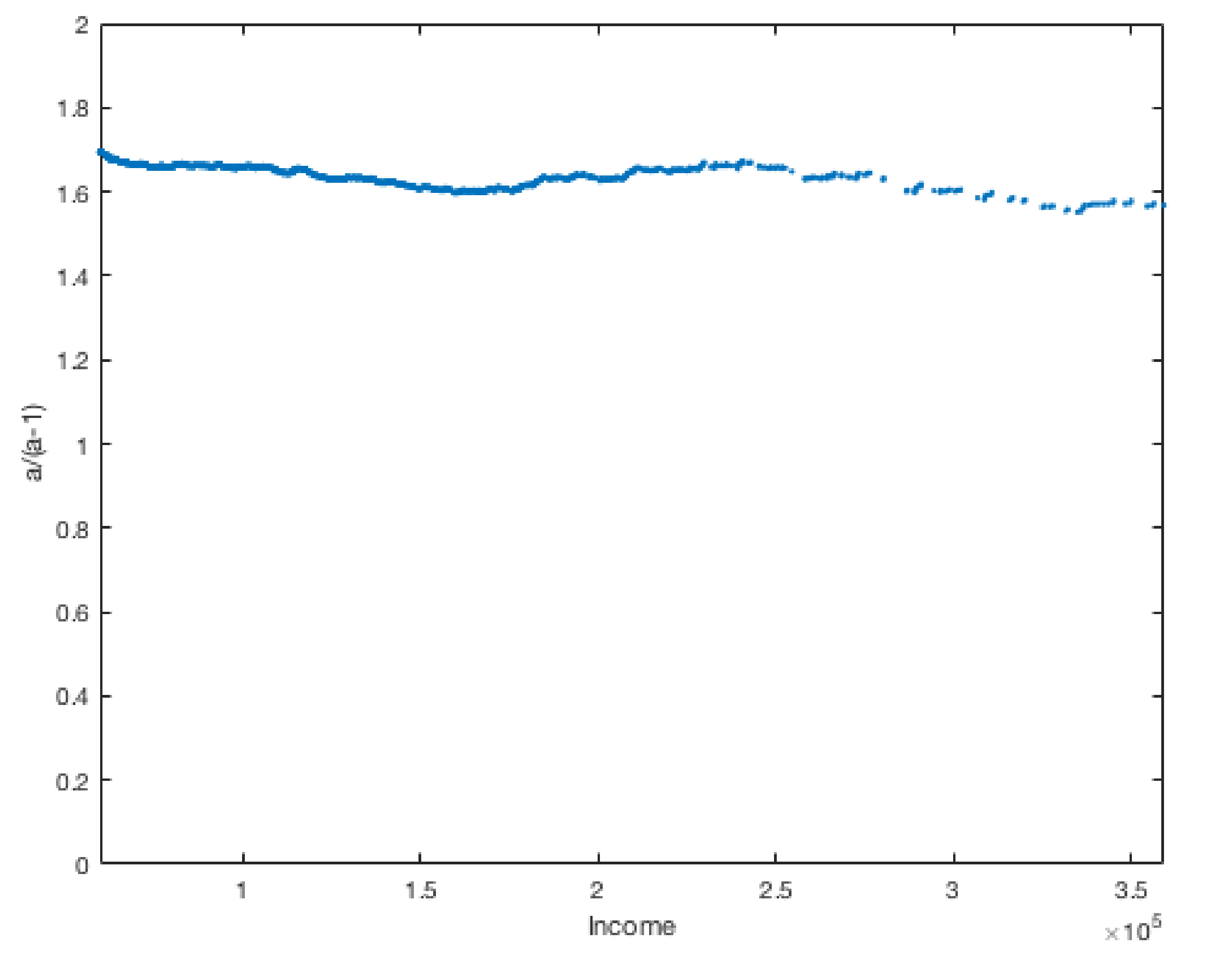
The value of $\frac{\bar{y}}{y^*}$ for income levels between 60,000 and 350,000

The ratio $\frac{\bar{y}}{y^*}$ is roughly equal to 1.6,
which implies $a = 2.66$,
and if $\bar{g} \approx 0$

$$\tau = \frac{1}{1 + 2.66\overline{\eta^u}}$$



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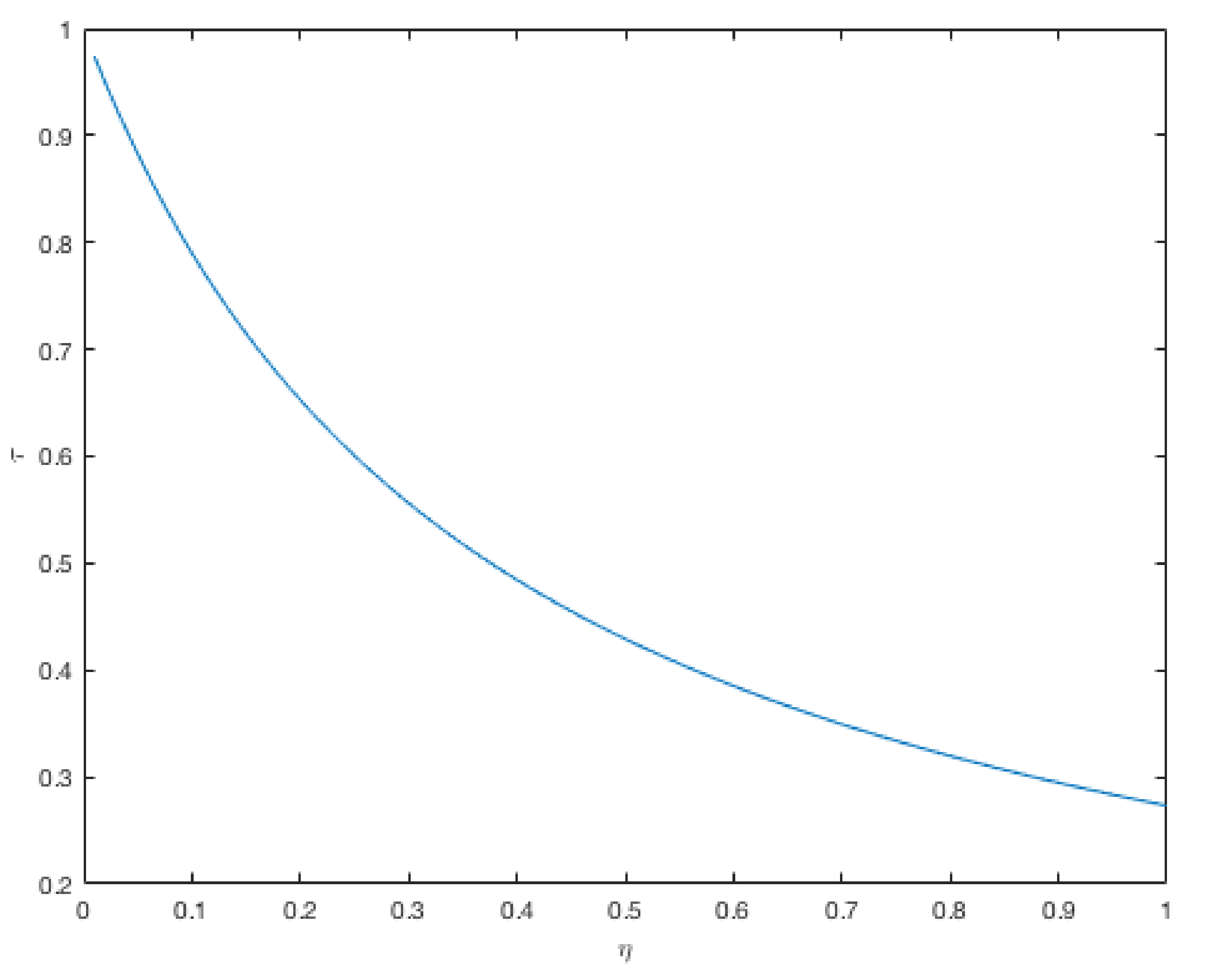
The optimal marginal tax rate as a function of η^u

Bargain *et al*. (2011) estimated the labor supply elasticity in Poland to be 0.1 (women) and 0.04 (men).
Then the optimal marginal tax rate in Poland for high-income earners would be 79% (women) and 90% (men).



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